

## Note on new massive gravity in $AdS_3$

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## Note on new massive gravity in $AdS_3$

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**ABSTRACT:** In this note we study the properties of linearized gravitational excitations in the new massive gravity theory in asymptotically  $AdS_3$  spacetime and find that there is also a critical point for the mass parameter at which massive gravitons become massless as in topological massive gravity in  $AdS_3$ . However, at this critical point in the new massive gravity the energy of all branches of highest weight gravitons vanish and the central charges also vanish within the Brown-Henneaux boundary conditions. The new massive gravity in asymptotically  $AdS_3$  spacetime seems to be trivial at this critical point under the Brown-Henneaux boundary conditions if the Brown-Henneaux boundary conditions can be consistent with this theory. At this point, the boundary conditions of log gravity may be preferred.

**KEYWORDS:** AdS-CFT Correspondence, Models of Quantum Gravity

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**Contents**

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>The new massive gravity theory</b>	<b>2</b>
<b>3</b>	<b>Gravitons in <math>AdS_3</math></b>	<b>4</b>
3.1	The equation of motion for graviton	4
3.2	Solutions of gravitons	5
<b>4</b>	<b>Positivity of energy</b>	<b>6</b>
<b>5</b>	<b>Conclusion and discussion</b>	<b>8</b>

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**1 Introduction**

The AdS/CFT correspondence [1–4] has given rise to increasing interest in the study of gravity theory in asymptotically  $AdS_3$  spacetime. In three dimensions, the Riemann tensor can be fully determined by the Ricci tensor and there are no locally propagating degrees of freedom in pure gravity theory. However, with a negative cosmological constant black hole solutions [5] can be found which obey the laws of black hole thermodynamics and possess a nonzero entropy. For pure gravity in  $AdS_3$  the CFT dual of this quantum gravity has been identified in [6, 7]. More discussions on  $AdS_3/CFT_2$  correspondence can be found in [8–18].

To develop further understanding to quantum gravity on asymptotically  $AdS_3$  spacetime, a deformation of pure Einstein gravity theory named topological massive gravity [19, 20] with a negative cosmological constant has been studied [21]. In this topological massive gravity with a negative cosmological constant, it was conjectured that there exists a chiral point at which the theory becomes chiral with only right-moving modes. Later it was proved that it is indeed chiral at the critical point [22, 23]. More discussions on this theory can be found in [24–39].

Recently in [40] a new kind of massive gravity has been discovered in three dimensions. In this new massive gravity, higher derivative terms are added to the Einstein Hilbert action and unlike in topological massive gravity, parity is preserved in this new massive gravity. This new massive gravity is equivalent to the Pauli-Fierz action for a massive spin-2 field at the linearized level in asymptotically Minkowski spacetime. In [41], the unitarity of this new massive gravity was examined and this new massive gravity is generalized to higher dimensions, but the unitarity is violated in higher dimensions. Warped AdS black hole solutions for this new massive gravity with a negative cosmological constant have been found in [42].

In this paper we study the behavior of the linearized gravitational excitations of this new massive gravity with a negative cosmological constant in the background of  $AdS_3$  spacetime with Brown-Henneaux boundary conditions. We find that similar to gravitons in topological massive gravity in  $AdS_3$  background, we also have a critical value of the mass parameter at which massive gravitons become massless. However, in this new massive gravity, the energy of both highest weight massless and massive gravitons are all zero and the central charge is also zero at the critical point. As long as the Brown-Henneaux boundary conditions can be consistent with this theory, the theory looks trivial at the critical point. But we need further studies on the consistency of the Brown-Henneaux boundary condition and the conserved charges associated with the symmetry.

In the remainder of this paper, we will first review the new massive gravity with a negative cosmological constant and write out the central charge in section 2. In section 3 we will calculate the behavior of the linearized gravitational excitations around  $AdS_3$ . In section 4 we show that to have an  $AdS_3$  vacuum we can only have a critical value of the mass parameter but at this special point the theory seems to be trivial. Section 5 is devoted to conclusions and discussions.

## 2 The new massive gravity theory

In this section we review the basics of the new massive gravity with a negative cosmological constant. The action of the new massive gravity theory can be written as<sup>1</sup>[40]

$$I = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left[ R - 2\lambda - \frac{1}{m^2} K \right], \tag{2.1}$$

where

$$K = R^{\mu\nu} R_{\mu\nu} - \frac{3}{8} R^2, \tag{2.2}$$

$m$  is the mass parameter of this massive gravity and  $\lambda$  is a constant which is different from the cosmological constant. The Einstein equation of motion of this action is

$$G_{\mu\nu} + \lambda g_{\mu\nu} - \frac{1}{2m^2} K_{\mu\nu} = 0 \tag{2.3}$$

where

$$K_{\mu\nu} = -\frac{1}{2} \nabla^2 R g_{\mu\nu} - \frac{1}{2} \nabla_\mu \nabla_\nu R + 2 \nabla^2 R_{\mu\nu} + 4 R_{\mu\alpha\nu\beta} R^{\alpha\beta} - \frac{3}{2} R R_{\mu\nu} - R_{\alpha\beta} R^{\alpha\beta} g_{\mu\nu} + \frac{3}{8} R^2 g_{\mu\nu}. \tag{2.4}$$

One special feature of this choice of  $K$  is that  $g^{\mu\nu} K_{\mu\nu} = K$ .

To have an asymptotically  $AdS_3$  solution, we have to introduce a non-zero  $\lambda$ . For an  $AdS_3$  solution

$$ds^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu = \ell^2 (-\cosh^2 \rho d\tau^2 + \sinh^2 \rho d\phi^2 + d\rho^2) \tag{2.5}$$

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<sup>1</sup> We take the metric signature  $(-,+,+)$  and follow the notation and conventions of MTW [46]. We assume  $m^2 > 0$  and  $G$  is the three dimensional Newton constant which is positive here.

with an AdS radius  $\ell$  which is related to the cosmological constant  $\Lambda$  by

$$\ell^{-2} = -\Lambda, \tag{2.6}$$

the Riemann tensor, Ricci tensor and Ricci scalar of the  $AdS_3$  are

$$\bar{R}_{\mu\alpha\nu\beta} = \Lambda(\bar{g}_{\mu\nu}\bar{g}_{\alpha\beta} - \bar{g}_{\mu\beta}\bar{g}_{\alpha\nu}), \quad \bar{R}_{\mu\nu} = 2\Lambda\bar{g}_{\mu\nu}, \quad \bar{R} = 6\Lambda, \tag{2.7}$$

and  $K_{\mu\nu} = -1/2\Lambda^2 g_{\mu\nu}$ . Thus the  $\lambda$  in the action should be related to the cosmological constant  $\Lambda$  and the mass parameter by

$$m^2 = \frac{\Lambda^2}{4(-\lambda + \Lambda)}. \tag{2.8}$$

Note here for a given  $\lambda < 0$ , there can be both AdS and de-Sitter solutions to this action. We only focus on the AdS solution.

The metric has an isometry group  $SL(2, R)_L \times SL(2, R)_R$ . The  $SL(2, R)_L$  generators are [21]

$$L_0 = i\partial_u, \tag{2.9}$$

$$L_{-1} = ie^{-iu} \left[ \frac{\cosh 2\rho}{\sinh 2\rho} \partial_u - \frac{1}{\sinh 2\rho} \partial_v + \frac{i}{2} \partial_\rho \right], \tag{2.10}$$

$$L_1 = ie^{iu} \left[ \frac{\cosh 2\rho}{\sinh 2\rho} \partial_u - \frac{1}{\sinh 2\rho} \partial_v - \frac{i}{2} \partial_\rho \right], \tag{2.11}$$

where  $u \equiv \tau + \phi, v \equiv \tau - \phi$ . The  $SL(2, R)_R$  generators  $\{\bar{L}_0, \bar{L}_{\pm 1}\}$  are given by the above expressions with  $u \leftrightarrow v$ . The central charge of this gravity theory in asymptotically  $AdS_3$  spacetime can be calculated using the formula [18, 43–45]

$$c = \frac{\ell}{2G} \bar{g}_{\mu\nu} \frac{\partial \mathcal{L}}{\partial R_{\mu\nu}}, \tag{2.12}$$

where  $\mathcal{L}$  is the Lagrangian density, to be

$$c = \frac{3\ell}{2G} \left( 1 - \frac{1}{2m^2\ell^2} \right). \tag{2.13}$$

Unlike topological massive gravity, because we have no Chern-Simons term here the left moving and right moving central charges are equal and we have

$$c_L = c_R = \frac{3\ell}{2G} \left( 1 - \frac{1}{2m^2\ell^2} \right). \tag{2.14}$$

We can calculate the entropy of BTZ black holes using Cardy formula as in [21] and we find that the entropy obtained from the Cardy formula is just the same as the one obtained in [42] using Wald's formula. In order to get a non-negative central charge we need to set  $m^2\ell^2 \geq 1/2$ . Here we see that  $m^2 < 0$  can also give positive central charge and entropy. However, in the next section we will see that  $m^2 < 0$  is not allowed. The mass of the BTZ black hole is also non-negative in this parameter region as can be found in [42].

### 3 Gravitons in $AdS_3$

In this section we analyze the behavior of linearized gravitational excitations on the background  $AdS_3$  spacetime in this new massive gravity theory.

#### 3.1 The equation of motion for graviton

We first give the equation of motion for gravitons in this subsection. By expanding

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \quad (3.1)$$

with  $h_{\mu\nu}$  small, we have the following physical quantities to the leading order

$$\Gamma_{\mu\nu}^{\lambda(1)} = \frac{1}{2}\bar{g}^{\lambda\alpha} (\bar{\nabla}_\mu h_{\alpha\nu} + \bar{\nabla}_\nu h_{\mu\alpha} - \bar{\nabla}_\alpha h_{\mu\nu}), \quad (3.2)$$

$$R_{\rho\nu\mu}^{\lambda(1)} = \frac{1}{2}\bar{g}^{\lambda\alpha} (\bar{\nabla}_\nu \bar{\nabla}_\mu h_{\alpha\rho} + \bar{\nabla}_\nu \bar{\nabla}_\rho h_{\mu\alpha} - \bar{\nabla}_\nu \bar{\nabla}_\alpha h_{\mu\rho}) - (\mu \leftrightarrow \nu), \quad (3.3)$$

$$R_{\mu\nu}^{(1)} = \frac{1}{2} (-\bar{\nabla}^2 h_{\mu\nu} - \bar{\nabla}_\mu \bar{\nabla}_\nu h + \bar{\nabla}^\sigma \bar{\nabla}_\nu h_{\sigma\mu} + \bar{\nabla}^\sigma \bar{\nabla}_\mu h_{\sigma\nu}), \quad (3.4)$$

$$R^{(1)} \equiv (R_{\mu\nu} g^{\mu\nu})^{(1)} = -\bar{\nabla}^2 h + \bar{\nabla}_\mu \bar{\nabla}_\nu h^{\mu\nu} - 2\Lambda h, \quad (3.5)$$

$$(\nabla_\mu \nabla_\nu R)^{(1)} = \bar{\nabla}_\mu \bar{\nabla}_\nu R^{(1)}, \quad (3.6)$$

$$(\nabla_\alpha \nabla_\beta R_{\mu\nu})^{(1)} = \bar{\nabla}_\alpha \bar{\nabla}_\beta R_{\mu\nu}^{(1)} - 2\Lambda \bar{\nabla}_\alpha \bar{\nabla}_\beta h_{\mu\nu}. \quad (3.7)$$

Substituting these quantities to the equation of motion (2.3), we have the equation of motion for graviton  $h_{\mu\nu}$  to be

$$G_{\mu\nu}^{(1)} + \lambda h_{\mu\nu} - \frac{1}{2m^2} K_{\mu\nu}^{(1)} = 0, \quad (3.8)$$

where

$$G_{\mu\nu}^{(1)} = R_{\mu\nu}^{(1)} - \frac{1}{2}\bar{g}_{\mu\nu} R^{(1)} - 3\Lambda h_{\mu\nu}, \quad (3.9)$$

$$K_{\mu\nu}^{(1)} = -\frac{1}{2}\bar{\nabla}^2 R^{(1)} \bar{g}_{\mu\nu} - \frac{1}{2}\bar{\nabla}_\mu \bar{\nabla}_\nu R^{(1)} + 2\bar{\nabla}^2 R_{\mu\nu}^{(1)} - 4\Lambda \bar{\nabla}^2 h_{\mu\nu} - 5\Lambda R_{\mu\nu}^{(1)} + \frac{3}{2}\Lambda R^{(1)} \bar{g}_{\mu\nu} + \frac{19}{2}\Lambda^2 h_{\mu\nu}. \quad (3.10)$$

Taking the trace of the equation of motion by multiplying  $\bar{g}^{\mu\nu}$  on both sides, we obtain

$$R^{(1)} = 0. \quad (3.11)$$

Then we fix the gauge as it was done in [21]. We define  $\tilde{h}_{\mu\nu} \equiv h_{\mu\nu} - \bar{g}_{\mu\nu} h$ , which gives  $\tilde{h} = -2h$  and

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu} \tilde{h}, \quad (3.12)$$

$$R^{(1)} = \bar{\nabla}_\mu \bar{\nabla}_\nu \tilde{h}^{\mu\nu} + \Lambda \tilde{h} = 0. \quad (3.13)$$

Thus the gauge

$$\bar{\nabla}_\mu \tilde{h}^{\mu\nu} = 0 \quad (3.14)$$

together with the linearized equation of motion implies tracelessness of  $h_{\mu\nu}$ :  $\tilde{h} = -2h = 0$ . This gauge is equivalent to the harmonic plus traceless gauge  $\bar{\nabla}_\mu h^{\mu\nu} = h = 0$ .

Noting that

$$[\bar{\nabla}_\sigma, \bar{\nabla}_\mu] h_\nu^\sigma = 3\Lambda h_{\mu\nu} - \Lambda h g_{\mu\nu} \quad (3.15)$$

and imposing the gauge condition, we obtain

$$R_{\mu\nu}^{(1)} = \frac{1}{2} (-\bar{\nabla}^2 h_{\mu\nu} + 6\Lambda h_{\mu\nu}). \quad (3.16)$$

By using  $R^{(1)} = 0$  and gauge fixing conditions  $\bar{\nabla}_\mu h^{\mu\nu} = h = 0$ , we reach the following simplified equation for graviton  $h_{\mu\nu}$ :

$$G_{\mu\nu}^{(1)} + \lambda h_{\mu\nu} - \frac{1}{2m^2} K_{\mu\nu}^{(1)} = 0, \quad (3.17)$$

with

$$G_{\mu\nu}^{(1)} = R_{\mu\nu}^{(1)} - 3\Lambda h_{\mu\nu} = -\frac{1}{2}\bar{\nabla}^2 h_{\mu\nu}, \quad (3.18)$$

$$K_{\mu\nu}^{(1)} = -\bar{\nabla}^2 \bar{\nabla}^2 h_{\mu\nu} + \frac{9}{2}\Lambda \bar{\nabla}^2 h_{\mu\nu} - \frac{11}{2}\Lambda^2 h_{\mu\nu}. \quad (3.19)$$

Thus the equation of motion for graviton could be factorized as

$$\left[ \bar{\nabla}^2 - \left( m^2 + \frac{5\Lambda}{2} \right) \right] \left[ \bar{\nabla}^2 - 2\Lambda \right] h_{\mu\nu} = 0. \quad (3.20)$$

From the equation of motion above, we can easily see that there are two branches of solutions. The first is

$$\left[ \bar{\nabla}^2 - 2\Lambda \right] h_{\mu\nu} = 0, \quad (3.21)$$

which corresponds to the modes of the left and right moving massless gravitons, and the second one is

$$\left[ \bar{\nabla}^2 - \left( m^2 + \frac{5\Lambda}{2} \right) \right] h_{\mu\nu} = 0, \quad (3.22)$$

which corresponds to massive sectors of gravitons. To have a non-negative mass, we also need to have  $m^2 \ell^2 \geq 1/2$  which coincides with the condition needed to ensure a non-negative central charge. To give a positive mass of the gravitons,  $m^2 < 0$  is not allowed. It can be seen that when  $m^2 \ell^2 = 1/2$ , the massive sector of gravitons becomes massless. It is very similar to chiral gravity at the critical point. In the next subsection we will solve the equation of motion of gravitons.

### 3.2 Solutions of gravitons

Using the expressions of the generators of the isometry group  $SL(2, R)_L \times SL(2, R)_R$  of  $AdS_3$ , we could easily simplify the equation of motion for gravitons using

$$\bar{\nabla}^2 h_{\mu\nu} = - \left[ \frac{2}{\ell^2} (L^2 + \bar{L}^2) + \frac{6}{\ell^2} \right] h_{\mu\nu} \quad (3.23)$$

to be

$$\left[-\frac{2}{\ell^2}(L^2 + \bar{L}^2) - \frac{7}{2\ell^2} - m^2\right] \left[\frac{1}{\ell^2}(L^2 + \bar{L}^2) + \frac{2}{\ell^2}\right] h_{\mu\nu} = 0. \quad (3.24)$$

We follow [21] to classify the solutions of (3.20) using the  $SL(2, R)_L \times SL(2, R)_R$  algebra. Considering highest weight states with weight  $(h, \bar{h})$  and using  $L^2|\psi_{\mu\nu}\rangle = -h(h-1)|\psi_{\mu\nu}\rangle$ , we obtain

$$\left[2h(h-1) + 2\bar{h}(\bar{h}-1) - \frac{7}{2} - m^2\ell^2\right] [h(h-1) + \bar{h}(\bar{h}-1) - 2] = 0, \quad h - \bar{h} = \pm 2 \quad (3.25)$$

for the highest weight states. There are two branches of solutions for this equation. The first one is the massless one  $h(h-1) + \bar{h}(\bar{h}-1) - 2 = 0$ , which gives

$$h = \frac{3 \pm 1}{2}, \bar{h} = \frac{-1 \pm 1}{2} \quad \text{or} \quad h = \frac{-1 \pm 1}{2}, \bar{h} = \frac{3 \pm 1}{2}. \quad (3.26)$$

The solutions with the lower sign will blow up at infinity as argued in [21], so we will only keep the upper ones corresponding to weights  $(2, 0)$  and  $(0, 2)$ . We will refer to these as left and right-moving massless gravitons.

The second branch has  $2h(h-1) + 2\bar{h}(\bar{h}-1) - \frac{7}{2} - m^2\ell^2 = 0$ , which gives

$$h = \frac{6 \pm \sqrt{2 + 4m^2\ell^2}}{4}, \bar{h} = \frac{-2 \pm \sqrt{2 + 4m^2\ell^2}}{4} \quad (3.27)$$

or

$$h = \frac{-2 \pm \sqrt{2 + 4m^2\ell^2}}{4}, \bar{h} = \frac{6 \pm \sqrt{2 + 4m^2\ell^2}}{4}. \quad (3.28)$$

The solutions with the lower sign will also blow up at the infinity, so we will only keep the ones with the upper sign which correspond to weights  $\left(\frac{6 + \sqrt{2 + 4m^2\ell^2}}{4}, \frac{-2 + \sqrt{2 + 4m^2\ell^2}}{4}\right)$  and  $\left(\frac{-2 + \sqrt{2 + 4m^2\ell^2}}{4}, \frac{6 + \sqrt{2 + 4m^2\ell^2}}{4}\right)$ . We refer to these modes as massive gravitons. Here we should also have  $m^2\ell^2 \geq 1/2$  in order that there is no blow up at the infinity. The point  $m^2\ell^2 = 1/2$  is very interesting, because the massive gravitons become massless at this point.<sup>2</sup>

## 4 Positivity of energy

In this section, we follow the method of [21] to calculate the energy of the linearized gravitons in  $AdS_3$  background. We assume Brown-Henneaux boundary conditions [47] in the following calculation. However, the consistency of this kind of condition with the new massive gravity still needs to be further confirmed.

The fluctuation  $h_{\mu\nu}$  can be decomposed as

$$h_{\mu\nu} = h_{\mu\nu}^M + h_{\mu\nu}^m, \quad (4.1)$$

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<sup>2</sup>At this critical point, there can be other solutions [26] of the graviton which, however, do not obey the Brown-Henneaux boundary conditions [35, 38, 39]. We would like to thank Wei Song for pointing this out to us.



and here we use "M" to denote "massive" modes and "m" to denote "massless" modes.

Up to total derivatives, the quadratic action of  $h_{\mu\nu}$  can be written as

$$\begin{aligned}
 S_2 &= -\frac{1}{32\pi G} \int d^3x \sqrt{-\bar{g}} h^{\mu\nu} \left( G_{\mu\nu}^{(1)} + \lambda h_{\mu\nu} - \frac{1}{2m^2} K_{\mu\nu}^{(1)} \right) \\
 &= -\frac{1}{32\pi G} \int d^3x \sqrt{-\bar{g}} \left\{ \frac{1}{2m^2} \bar{\nabla}^2 h^{\mu\nu} \bar{\nabla}^2 h_{\mu\nu} + \left( \frac{1}{2} - \frac{9}{4m^2\ell^2} \right) \bar{\nabla}^\lambda h^{\mu\nu} \bar{\nabla}_\lambda h_{\mu\nu} \right. \\
 &\quad \left. + \left( \frac{11}{4m^2\ell^4} + \lambda \right) h^{\mu\nu} h_{\mu\nu} \right\}.
 \end{aligned} \tag{4.2}$$

The momentum conjugate to  $h_{\mu\nu}$  is

$$\Pi^{(1)\mu\nu} = \frac{\sqrt{-\bar{g}}}{32\pi G} \left\{ \frac{1}{m^2} \bar{\nabla}^0 \bar{\nabla}^2 h^{\mu\nu} - \left( 1 - \frac{9}{2m^2\ell^2} \right) \bar{\nabla}^0 h^{\mu\nu} \right\}, \tag{4.3}$$

and using the equations of motion we can have

$$\Pi_m^{(1)\mu\nu} = -\frac{\sqrt{-\bar{g}}}{32\pi G} \left( 1 - \frac{5}{2m^2\ell^2} \right) \bar{\nabla}^0 h_m^{\mu\nu}, \tag{4.4}$$

$$\Pi_M^{(1)\mu\nu} = \frac{\sqrt{-\bar{g}}}{32\pi G} \left( \frac{2}{m^2\ell^2} \right) \bar{\nabla}^0 h_M^{\mu\nu}. \tag{4.5}$$

Because we have up to four time derivatives in the Lagrangian, using the Ostrogradsky method [21, 48] we should also introduce  $K_{\mu\nu} \equiv \bar{\nabla}_0 h_{\mu\nu}$  as a canonical variable, whose conjugate momentum is

$$\Pi^{(2)\mu\nu} = -\frac{\sqrt{-\bar{g}}}{32\pi G} \frac{\bar{g}^{00}}{m^2} \bar{\nabla}^2 h^{\mu\nu}, \tag{4.6}$$

and again using equations of motion we can have

$$\Pi_m^{(2)\mu\nu} = \frac{\sqrt{-\bar{g}}}{32\pi G} \frac{2\bar{g}^{00}}{m^2\ell^2} h_m^{\mu\nu}, \tag{4.7}$$

$$\Pi_M^{(2)\mu\nu} = -\frac{\sqrt{-\bar{g}}}{32\pi G} \bar{g}^{00} \left( 1 - \frac{5}{2m^2\ell^2} \right) h_M^{\mu\nu}. \tag{4.8}$$

The Hamiltonian is then expressed using these variables to be

$$H = \int d^2x \left( \dot{h}_{\mu\nu} \Pi^{(1)\mu\nu} + \dot{K}_{\mu\nu} \Pi^{(2)\mu\nu} - \mathcal{L} \right). \tag{4.9}$$

After substituting the equations of motion of the highest weight states, we can have the energies as follows

$$E_m = -\left( 1 - \frac{1}{2m^2\ell^2} \right) \int d^2x \frac{\sqrt{-\bar{g}}}{32\pi G} \left\{ \bar{\nabla}^0 h_m^{\mu\nu} \dot{h}_{m\mu\nu} \right\}, \tag{4.10}$$

$$E_M = \left( 1 - \frac{1}{2m^2\ell^2} \right) \int d^2x \frac{\sqrt{-\bar{g}}}{32\pi G} \left\{ \bar{\nabla}^0 h_M^{\mu\nu} \dot{h}_{M\mu\nu} \right\}. \tag{4.11}$$

By plugging in the solutions of highest weight gravitons [21] with our  $(h, \bar{h})$  into the above expression, we find that for the massless modes  $h_{\mu\nu}^m$ , the energy is positive for  $m^2\ell^2 >$

1/2 and negative for  $m^2\ell^2 < 1/2$ ; for the massive modes  $h_{\mu\nu}^M$ , the energy is positive for  $m^2\ell^2 < 1/2$  and negative for  $m^2\ell^2 > 1/2$ . The same as in chiral gravity, to have an asymptotically  $AdS_3$  vacuum, we can only have  $m^2\ell^2 = 1/2$ . However, at this point, the energies of both highest weight massless and massive modes are zero, and both the left moving and right moving central charges are zero. This can be seen as a sign that the new massive gravity in asymptotically  $AdS_3$  may be trivial at this critical point and both the massless and massive gravitons can be viewed as pure gauge in the framework of Brown-Henneaux boundary conditions. If they are not pure gauge, the  $AdS_3$  vacuum at this critical point is not stable.

However, there are several points to note here. First, we have assumed the Brown-Henneaux boundary conditions through the calculations, but the consistency of these conditions with the new massive gravity still need to be confirmed, as it has been done in [25] for topological massive gravity with a negative cosmological constant. Also even if the consistency is satisfied we still need to calculate the conserved charges to see if the charges are all zero at the critical point. Second, even if the above argument is satisfied, this does not mean that the new massive gravity with a negative cosmological constant is trivial because we may still have another stable vacuum which is not  $AdS_3$  for other values of the mass parameter, like the warped  $AdS_3$  black holes in topological massive gravity. Also there may be other ways to calculate the energy of the gravitons which do not give this result.

## 5 Conclusion and discussion

In this note, we studied the new massive gravity theory in asymptotically  $AdS_3$  spacetime, and found that this theory could only be sensible at  $m^2\ell^2 = 1/2$  in order to have an  $AdS_3$  vacuum. However, at this special point, the energies of both the highest weight massless and massive modes are zero, and both the left moving and right moving central charges are zero. The new massive gravity in asymptotically  $AdS_3$  may be trivial at this critical point in the sense of the Brown-Henneaux boundary conditions. At this point, the boundary conditions of log gravity [26, 35, 38, 39] may be preferred and give interesting results.

In the calculations we have assumed the Brown-Henneaux boundary conditions, but we need to further analyze the consistency of the boundary condition with the new massive gravity. Also we need to check if the conserved charges are zero at the critical point. Even if this theory is indeed trivial at the critical point we can have another stable vacuum which is not locally  $AdS_3$  for any value of the mass parameter. And we can also relax the boundary conditions to have more interesting physics in this theory like it was done in [26, 35, 38, 39].

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## References

- [1] J.M. Maldacena, *The large- $N$  limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [[hep-th/9711200](#)] [[SPIRES](#)].
- [2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett. B* **428** (1998) 105 [[hep-th/9802109](#)] [[SPIRES](#)].
- [3] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [[hep-th/9802150](#)] [[SPIRES](#)].
- [4] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large- $N$  field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [[hep-th/9905111](#)] [[SPIRES](#)].
- [5] M. Bañados, C. Teitelboim and J. Zanelli, *The black hole in three-dimensional space-time*, *Phys. Rev. Lett.* **69** (1992) 1849 [[hep-th/9204099](#)] [[SPIRES](#)].
- [6] E. Witten, *(2 + 1)-dimensional gravity as an exactly soluble system*, *Nucl. Phys. B* **311** (1988) 46 [[SPIRES](#)].
- [7] E. Witten, *Three-dimensional gravity revisited*, [arXiv:0706.3359](#) [[SPIRES](#)].
- [8] J. Manschot, *AdS<sub>3</sub> partition functions reconstructed*, *JHEP* **10** (2007) 103 [[arXiv:0707.1159](#)] [[SPIRES](#)].
- [9] D. Gaiotto and X. Yin, *Genus two partition functions of extremal conformal field theories*, *JHEP* **08** (2007) 029 [[arXiv:0707.3437](#)] [[SPIRES](#)].
- [10] M.R. Gaberdiel, *Constraints on extremal self-dual CFTs*, *JHEP* **11** (2007) 087 [[arXiv:0707.4073](#)] [[SPIRES](#)].
- [11] S.D. Avramis, A. Kehagias and C. Mattheopoulou, *Three-dimensional AdS gravity and extremal CFTs at  $c = 8m$* , *JHEP* **11** (2007) 022 [[arXiv:0708.3386](#)] [[SPIRES](#)].
- [12] X. Yin, *Partition functions of three-dimensional pure gravity*, [arXiv:0710.2129](#) [[SPIRES](#)].
- [13] X. Yin, *On non-handlebody instantons in 3D gravity*, *JHEP* **09** (2008) 120 [[arXiv:0711.2803](#)] [[SPIRES](#)].
- [14] A. Maloney and E. Witten, *Quantum gravity partition functions in three dimensions*, [arXiv:0712.0155](#) [[SPIRES](#)].
- [15] J. Manschot and G.W. Moore, *A modern farey tail*, [arXiv:0712.0573](#) [[SPIRES](#)].
- [16] D. Gaiotto, *Monster symmetry and extremal CFTs*, [arXiv:0801.0988](#) [[SPIRES](#)].
- [17] S. Giombi, A. Maloney and X. Yin, *One-loop partition functions of 3D gravity*, *JHEP* **08** (2008) 007 [[arXiv:0804.1773](#)] [[SPIRES](#)].
- [18] P. Kraus, *Lectures on black holes and the AdS<sub>3</sub>/CFT<sub>2</sub> correspondence*, *Lect. Notes Phys.* **755** (2008) 193 [[hep-th/0609074](#)] [[SPIRES](#)].
- [19] S. Deser, R. Jackiw and S. Templeton, *Topologically massive gauge theories*, *Ann. Phys.* **140** (1982) 372 [*Erratum ibid.* **185** (1988) 406] [*Annals Phys.* **281** (2000) 409] [[SPIRES](#)].

- [20] S. Deser, R. Jackiw and S. Templeton, *Three-dimensional massive gauge theories*, *Phys. Rev. Lett.* **48** (1982) 975 [SPIRES].
- [21] W. Li, W. Song and A. Strominger, *Chiral gravity in three dimensions*, *JHEP* **04** (2008) 082 [arXiv:0801.4566] [SPIRES].
- [22] A. Strominger, *A simple proof of the chiral gravity conjecture*, arXiv:0808.0506 [SPIRES].
- [23] S. Carlip, *The constraint algebra of topologically massive AdS gravity*, *JHEP* **10** (2008) 078 [arXiv:0807.4152] [SPIRES].
- [24] S. Carlip, S. Deser, A. Waldron and D.K. Wise, *Cosmological topologically massive gravitons and photons*, *Class. Quant. Grav.* **26** (2009) 075008 [arXiv:0803.3998] [SPIRES].
- [25] K. Hotta, Y. Hyakutake, T. Kubota and H. Tanida, *Brown-Henneaux's canonical approach to topologically massive gravity*, *JHEP* **07** (2008) 066 [arXiv:0805.2005] [SPIRES].
- [26] D. Grumiller and N. Johansson, *Instability in cosmological topologically massive gravity at the chiral point*, *JHEP* **07** (2008) 134 [arXiv:0805.2610] [SPIRES].
- [27] W. Li, W. Song and A. Strominger, *Comment on 'Cosmological topological massive gravitons and photons'*, arXiv:0805.3101 [SPIRES].
- [28] M.-i. Park, *Constraint dynamics and gravitons in three dimensions*, *JHEP* **09** (2008) 084 [arXiv:0805.4328] [SPIRES].
- [29] D. Grumiller, R. Jackiw and N. Johansson, *Canonical analysis of cosmological topologically massive gravity at the chiral point*, arXiv:0806.4185 [SPIRES].
- [30] S. Carlip, S. Deser, A. Waldron and D.K. Wise, *Topologically massive AdS gravity*, *Phys. Lett.* **B 666** (2008) 272 [arXiv:0807.0486] [SPIRES].
- [31] G.W. Gibbons, C.N. Pope and E. Sezgin, *The general supersymmetric solution of topologically massive supergravity*, *Class. Quant. Grav.* **25** (2008) 205005 [arXiv:0807.2613] [SPIRES].
- [32] D. Anninos, W. Li, M. Padi, W. Song and A. Strominger, *Warped AdS<sub>3</sub> black holes*, *JHEP* **03** (2009) 130 [arXiv:0807.3040] [SPIRES].
- [33] G. Giribet, M. Kleban and M. Porrati, *Topologically massive gravity at the chiral point is not chiral*, *JHEP* **10** (2008) 045 [arXiv:0807.4703] [SPIRES].
- [34] G. Compere and S. Detournay, *Semi-classical central charge in topologically massive gravity*, *Class. Quant. Grav.* **26** (2009) 012001 [arXiv:0808.1911] [SPIRES].
- [35] D. Grumiller and N. Johansson, *Consistent boundary conditions for cosmological topologically massive gravity at the chiral point*, *Int. J. Mod. Phys.* **D 17** (2009) 2367 [arXiv:0808.2575] [SPIRES].
- [36] A. Garbarz, G. Giribet and Y. Vasquez, *Asymptotically AdS<sub>3</sub> solutions to topologically massive gravity at special values of the coupling constants*, *Phys. Rev.* **D 79** (2009) 044036 [arXiv:0811.4464] [SPIRES].
- [37] M. Blagojevic and B. Cvetkovic, *Canonical structure of topologically massive gravity with a cosmological constant*, arXiv:0812.4742 [SPIRES].
- [38] M. Henneaux, C. Martinez and R. Troncoso, *Asymptotically anti-de Sitter spacetimes in topologically massive gravity*, arXiv:0901.2874 [SPIRES].
- [39] A. Maloney, W. Song and A. Strominger, *Chiral gravity, log gravity and extremal CFT*, arXiv:0903.4573 [SPIRES].

- [40] E.A. Bergshoeff, O. Hohm and P.K. Townsend, *Massive gravity in three dimensions*, [arXiv:0901.1766](#) [[SPIRES](#)].
- [41] M. Nakasone and I. Oda, *On unitarity of massive gravity in three dimensions*, [arXiv:0902.3531](#) [[SPIRES](#)].
- [42] G. Clement, *Warped AdS<sub>3</sub> black holes in new massive gravity*, [arXiv:0902.4634](#) [[SPIRES](#)].
- [43] H. Saida and J. Soda, *Statistical entropy of BTZ black hole in higher curvature gravity*, *Phys. Lett. B* **471** (2000) 358 [[gr-qc/9909061](#)] [[SPIRES](#)].
- [44] P. Kraus and F. Larsen, *Microscopic black hole entropy in theories with higher derivatives*, *JHEP* **09** (2005) 034 [[hep-th/0506176](#)] [[SPIRES](#)].
- [45] M.-I. Park, *BTZ black hole with higher derivatives, the second law of thermodynamics and statistical entropy*, *Phys. Rev. D* **77** (2008) 126012 [[hep-th/0609027](#)] [[SPIRES](#)];  
*Can Hawking temperatures be negative?*, *Phys. Lett. B* **663** (2008) 259 [[hep-th/0610140](#)] [[SPIRES](#)];  
*Thoughts on the area theorem*, *Class. Quant. Grav.* **25** (2008) 095013 [[hep-th/0611048](#)] [[SPIRES](#)].
- [46] C.W. Misner, K.S. Thorne and J.A. Wheeler, *Gravitation*, San Francisco (1973), pg. 1279 [[SPIRES](#)].
- [47] J.D. Brown and M. Henneaux, *Central charges in the canonical realization of asymptotic symmetries: an example from three-dimensional gravity*, *Commun. Math. Phys.* **104** (1986) 207 [[SPIRES](#)].
- [48] I.L. Buchbinder, S.L. Lyahovich and V.A. Krychtin, *Canonical quantization of topologically massive gravity*, *Class. Quant. Grav.* **10** (1993) 2083 [[SPIRES](#)].